Exploring Light Amplification by Stimulated Emission in Lasers

Notes for the Instructor

Michael A. Waxman
Department of Chemistry
University of Wisconsin-Superior
Superior, WI 54880-2898

In a beginning physical chemistry course, the instructor might want to provide the students with a basic understanding of how the intensity of a laser beam depends upon the laser parameters, such as the length of the active medium or the reflectivity of the cavity mirrors. This goal is not as simple as it might seem to a student. It often looks to a novice that the greater the fraction of radiation escaping from the laser cavity outward, the higher is the power of the laser output. Reality turns out to be more complex, as students will learn from this module.

Generally speaking, the dynamics of laser generation is studied by solving a set of three coupled differential equations that describe the time evolution of the electric field inside the cavity, the polarization of the active medium, and the population inversion in it. The rationale for exercising this rigorous approach in a first physical chemistry course for the chemistry majors is questionable. Instead, in this document we use a very simple model that allows the students to grasp the essentials of the topic. We briefly describe the limitations of this model and suggest further readings at the end of the document.

One of the ideas we want the students to comprehend is the strong dependence of the light intensity in the cavity on the difference between light amplification and losses. The solution to the main problem posted in the Mathcad document entitled "Exploring Light Amplification by Stimulated Emission in Lasers" is the following one:

$$I_{las} = pI_0\left[(1-p)\exp(2aL)\right]^{\frac{ct}{2L}} \qquad (1)$$

Here *p* is the fraction of the power that is let through the mirror; I_0 is the initial intensity of radiation; *a* is the amplification coefficient of the active medium; *L* is the length of the cavity; *c* is the speed of light in the cavity; and *t* is the time elapsed. For a mirror letting 10% of the radiation through, equation (1) yields the value of 1.098 for the term in the square brackets, while for the mirror letting 20% of the radiation through this term is 0.977. Since the term in the square brackets is raised to the power of ct/2L=150, the first value yields the factor (for pI_0 to be multiplied by) of 1.2x10^6 and the second one about 0.03. Thus the students can come to see that if gain exceeds losses then the power output of a laser system would rise quickly; but if the losses (through the mirror in our example) exceed gain even a bit then there will be essentially no output power. Exercises 1-3 were designed to reinforce the notion of power output. In Exercise 1, students use simple calculus to find the following expression for the optimal value of p:

$$p = \frac{1}{\frac{ct}{2L}+1} \qquad (2)$$

They then proceed to check their predictions using Mathcad, and to explain their results physically. Exercise 4 begins the series of writing assignments that are embedded in the document. The dependencies in Exercise 5 (a-c) to be studied with Mathcad are all monotonic. This monotonicity can be easily understood from the same "gain vs. losses" argument. Students should be encouraged to develop this idea. Exercise 6 is intended to persuade the students to think about the limitations of the model used in this document.

The main "trick" in solving the design problem (Exercise 8) is to optimize the coefficient p for the given values of c, t, and L (which is not asked explicitly). From Eq. (2) above, we find p optimal ~ 0.0066 and thus a = 0.066 m^-1.

In the "Reflections" section, we reiterate the main idea to be taken from the document, stress the limitations of the model used, and invite the students to familiarize themselves with a standard, rigorous analysis from the reference source.